Classical Formulation of Light Deflection

Technical Paper: Classical Mechanics Reformulation of Gravitational Light Deflection

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1. Introduction

This document presents a fully classical-mechanics-based reformulation of gravitational light deflection. The framework is constructed intentionally without invoking General Relativity, spacetime curvature, tensor geometry, or relativistic geodesics. Instead, using only Newtonian potential theory, variational principles, and effective mechanics, we reproduce the exact General Relativity result for the light bending angle:

$$\alpha = \frac{4GM}{bc^2}$$

The objective is to demonstrate that the gravitational deflection of light can be reconstructed in a purely Newtonian language by introducing an effective potential and an effective Lagrangian for photons.

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2. Fundamental Classical Principles

We begin with core Newtonian concepts: * Total energy of a particle: $E = T + U$. * Lagrangian: $L = T - U$. * Newtonian gravitational potential: $U = -\frac{GMm}{r}$.

For massive particles, Newtonian dynamics yield the classical deflection angle:

$$\alpha_{\text{Newton}} = \frac{2GM}{bc^2}$$

which is exactly half of the correct GR value.

The classical model presented here modifies only the effective potential that light experiences, retaining all Newtonian machinery intact.

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3. Principle of Zero Total Energy for Light

A photon travels at constant speed c and cannot exchange kinetic energy in the Newtonian sense. We therefore impose:

$$E = 0 \implies T = -U$$

This condition corresponds to motion on the escape-energy threshold familiar from Newtonian dynamics, but here applied to light.

Substituting this into the classical Lagrangian gives:

$$L = T - U = -U - U = -2U$$

Thus the effective Lagrangian for light becomes:

$$L_{\text{light}} = 2U$$

This is not the physical Lagrangian of GR (which is identically zero for null geodesics), but a classical effective Lagrangian that reproduces the correct bending.

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4. Effective Gravitational Potential for Light

To match the observed bending, the attractive potential felt by light must be twice as strong as the Newtonian potential felt by slow-moving matter.

Define the effective potential as:

$$U_{\text{light}}(r) = -\frac{2GM}{rc^2}$$

This potential contains the same $1/r$ radial dependence as Newtonian gravity, but scaled by the dimensionless factor $2/c^2$.

This scaling is the key feature that makes the classical bending angle identical to the GR prediction.

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5. Classical Effective Force on Light

The force corresponding to the effective potential is:

$$F = -\frac{dU_{\text{light}}}{dr} = \frac{2GM}{c^2 r^2}$$

We interpret this as an effective acceleration field acting on light rays:

$$g_{\text{light}}(r) = \frac{2GM}{c^2 r^2}$$

This expression differs from Newtonian gravity only by the factor $2/c^2$, representing the enhanced coupling of light to the gravitational field.

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6. Integration of Trajectory and Bending Angle

Using the standard scattering geometry of a particle passing a point mass M with impact parameter b, and treating the deflection as the integrated transverse acceleration over the interaction time, we obtain:

$$\alpha = \int g_{\text{light}}(t)dt = \frac{4GM}{bc^2}$$

This is exactly the General Relativity prediction for the deflection of light by the Sun, first confirmed by Eddington's 1919 solar eclipse observations.

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7. Interpretation and Physical Meaning

The doubling of potential corresponds to the fact that in GR, both the time-time component and the radial component of the Schwarzschild metric contribute equally to deflection. In the classical formalism presented here:

- The enhanced potential U_{light} encapsulates these two contributions in a single Newtonian-like scalar potential.

- The effective acceleration g_{light} reproduces the GR geodesic curvature.
- The Lagrangian $L = 2U$ is a classical surrogate for the null-geodesic condition $ds^2 = 0$.

Thus, the entire gravitational bending phenomenon is encoded into a modified Newtonian potential without introducing curved spacetime explicitly.

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8. Conclusion

Using only Newtonian mechanics, energy conservation, and variational principles, we have constructed a classical effective model that: * Assigns an effective potential twice as strong as the Newtonian gravitational potential. * Yields an effective Lagrangian $L = 2U$ for photons. * Produces the exact GR deflection angle $\alpha = \frac{4GM}{bc^2}$.

This formulation provides a bridge between Newtonian intuition and relativistic predictions, offering a classical reinterpretation of light bending that may serve as a pedagogical or conceptual alternative to the standard GR derivation.

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9. References

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3. Dyson, F., Eddington, A., and Davidson, C., "A Determination of the Deflection of Light," Phil. Trans. Roy. Soc. (1920).
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